New lower bounds for the critical bond percolation models? two-point functions in any dimension.

Emma Bailey, Shirshendu Chatterjee, and Jack Hanson

Abstract

We study the nearest-neighbor bond percolation models on \mathbb{Z}^d and obtain new lower bounds, which hold for any dimension, on their two-point functions at criticality. The asymptotics of the two-point function at criticality is well known for d = 2 and d large enough. However, for the intermediate dimensions, e.g., for d = 3, the best lower bound that is known is the "Simon-Loeb" bound. In this work, we show that the Simon-Lieb bound is not sharp in any dimension. More precisely, the ratio of the two-point function and the Simon-Lieb bound is larger than a poly-log factor. Thus, we obtain a new lower bound for the two-point function at criticality for the intermediate dimensions including d = 3.

Keywords ---- bond percolation, criticality, two-point function